

# CAsoft User Manual

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## 1 Foreword

CASoft is a software developed as part of EMPIR 17SIP05 CASoft project, with the participation of LNE (France), NPL (United Kingdom) and RISE (Sweden). It aims at facilitating the use of the methodology provided by JCGM106:2012 [3] for dealing with measurement uncertainty in decision-making in conformity assessment. As a result, the definitions and concepts from JCGM106:2012 [3] apply.

## 2 Introduction

CASoft is a software dedicated to the implementation of the calculations for decision-making in conformity assessment. It is developed according to the guidelines from JCGM106:2012 [3]. As a result, it allows the calculation of conformance probabilities (and consequently of specific risks) and of global risks from information provided by the user about the tolerance interval, the acceptance interval and prior and measurement distributions.

In cases where multiple quantities are considered for decision-making, it is possible to use the so-called "multivariate" function of the software in which the input information are simplified with no prior knowledge available and only the choice between a Gaussian or a Student's t distribution for the measurement result. This was identified in the development team as an important need for the end user.

## 3 Installation and system requirement

CASoft has been developed for Windows operating computers. It requires a valid Microsoft Excel License in order to use the import/export functions. CASoft is an executable developed with MATLAB. Its use is free and does not require any purchase of a MATLAB licence. However, before the first use of CASoft on a computer, a MATLAB Component Runtime (MCR) needs to be downloaded and installed. This MCR can be found at :

<https://mathworks.com/products/compiler/matlab-runtime.html>.

Running CASoft requires version R2019b (9.7) for Windows environment.

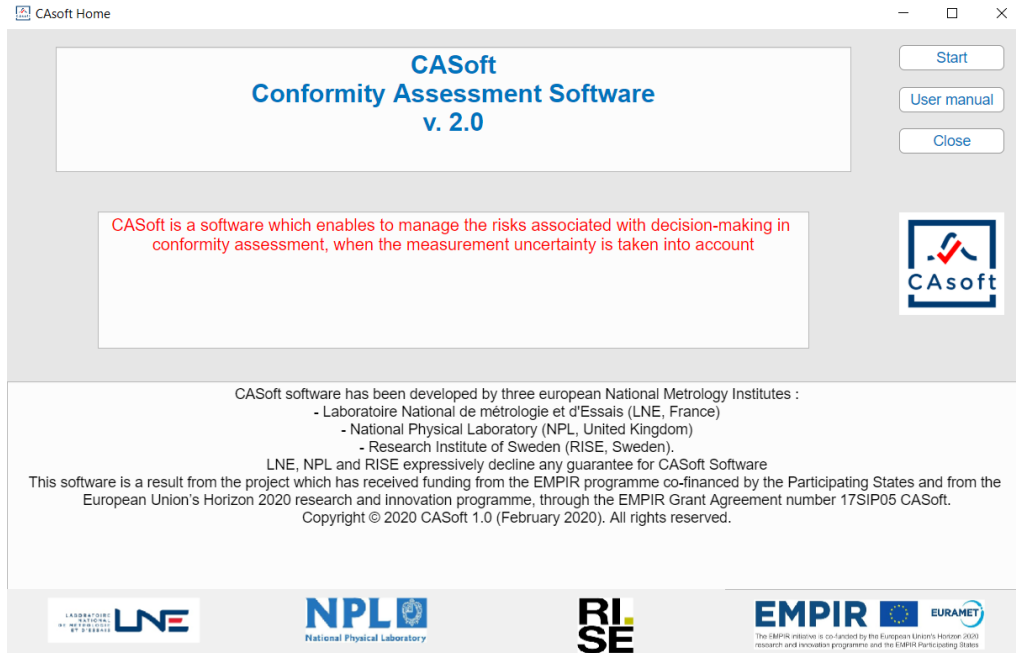


Figure 1: Homescreen of CASoft

## 4 Quick start guide

### 4.1 Home screen

The home screen of CASoft summarizes the objective of the software, acknowledges EURAMET for funding and contains a disclaimer. It allows to access the user manual and to open the main window by clicking the button "Start". Figure 1 provides a representation of this screen.

### 4.2 Input information

The Main window appears when the button 'Start' has been clicked by the user. It shows the title of software with :

- a button "Import file", which enables to open an EXCEL spreadsheet
- a 'Reset' button which enables to restart the software (new opening of the main window)



- Three buttons which enable to choose whether the case is a univariate case, a bivariate case or a case with "multiple characteristics". This latter choice has been designed to provide a tool that allows calibration and testing laboratories to compute conformance probabilities in an efficient way, with a simplified framework (no prior knowledge and only the consideration of Gaussian or t-Student distributions for the measurement result)

As a result, there are two ways for providing input information, either from an EXCEL file imported from the file system or directly through the navigation in CAsoft. Next section describes the steps for providing input information using the software.

### 4.3 Sequence of actions

Figure 2 summarizes the sequence of actions required by the user to obtain the expected results with CAsoft.

First, the dimension of the problem has to be specified. The dimension is either univariate (in this case, the problem is within the scope of JCGM106:2012 [3]), bivariate ([4]) or multivariate (Figure 3).

In the univariate case, the user is asked to specify the tolerance interval (Figure 4), which may be :

- two-sided : the entity is compared with both a lower and upper specification limits
- one-sided with a lower tolerance bound : the entity should be higher than a single lower tolerance limit
- one-sided with an upper tolerance bound : the entity should be lower than a single upper tolerance limit

Then, the nature of the calculation has to be specified. This calculation may be a conformance probability, an acceptance interval or global risks:

- Conformance probability: the probability distributions functions (PDF) for the prior knowledge (if available) and for the measurement have to be specified. Such distributions are respectively denoted as  $g_0(\eta)$  and  $h(\eta_m|\eta)$  in JCGM106:2012.

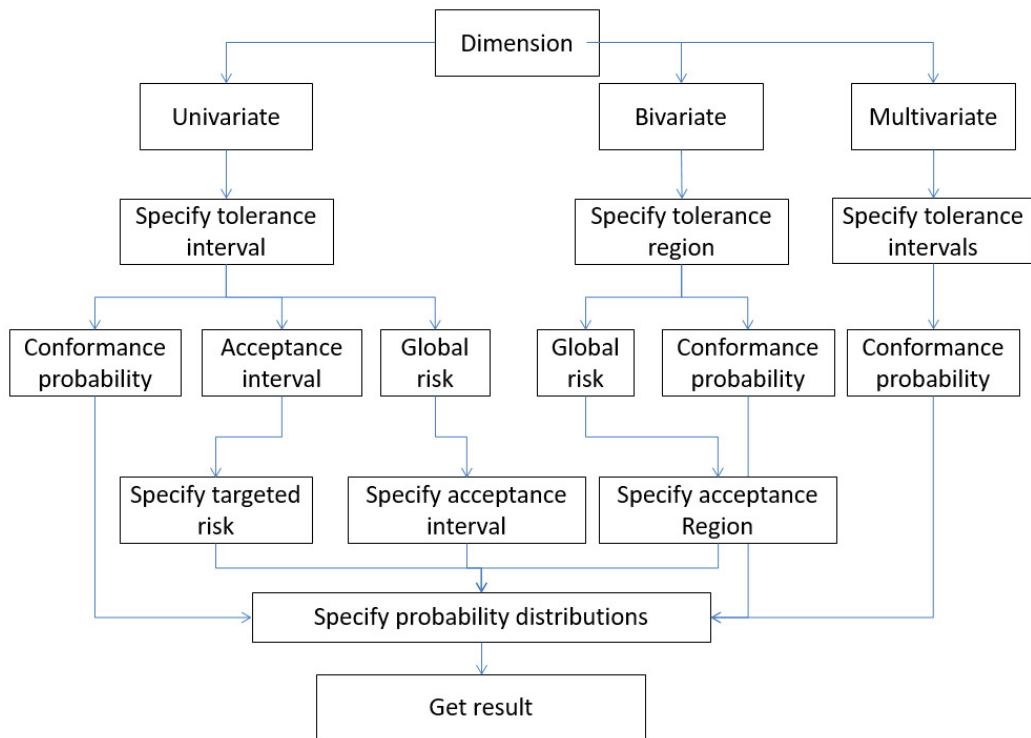


Figure 2: Sequence of actions in CAsoft, from the specification of input information to the result of the calculation

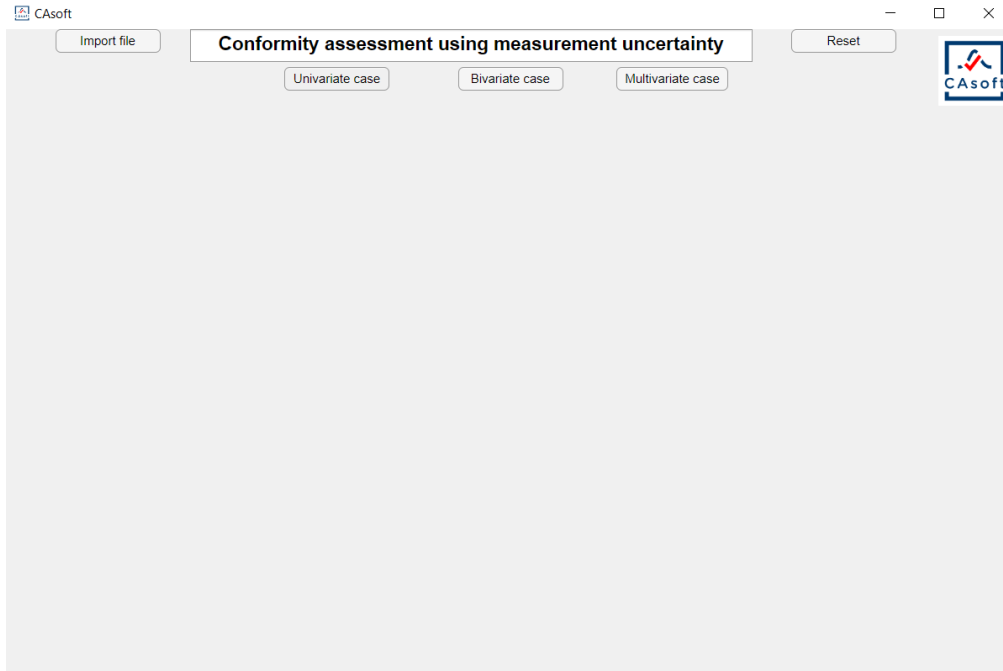


Figure 3: Main window of CAsoft : the first step is to select the dimension of the case. Univariate is for the case where a single measurand is considered for decision-making whereas Bivariate should be selected if the decision is made on the basis of two measurands and Multivariate should be selected to deal with more than 2 quantities independently

Specification of the tolerance interval			Specification of the tolerance interval	
Tolerance interval	Two-Sided ▼		Tolerance interval	Two-Sided ▼
	Y1	Y2		Y1
Lower bound	<input type="text" value="2"/>	<input type="text" value="1"/>	Lower bound	<input type="text" value="2"/>
Upper bound	<input type="text" value="4"/>	<input type="text" value="6"/>	Upper bound	<input type="text" value="4"/>

Figure 4: Specification of a tolerance region (left) or a tolerance interval (right)



- Acceptance interval : The determination of an optimal acceptance interval requires the definition of a targeted risk (either "consumer's" or "producer's" and a targeted value for this risk). Then a prior PDF  $g_0(\eta)$  and a PDF for the measurement  $h(\eta_m|\eta)$  have to be specified.
- Global risks: The determination of the global risks requires the specification of acceptance intervals. Then a prior PDF  $g_0(\eta)$  and a PDF for the measurement  $g_0(\eta)$  have to be specified.

In the bivariate case, the first input data consists in the specification of the tolerance region (Figure 4). For simplicity purposes, this region is supposed to be "rectangular", i.e. the tolerance bounds for each component do not depend on the tolerance limits for the other component.

Two methods are available in the bivariate case: conformance probabilities and global risks. For each methods, the required inputs are then a generalization of the univariate case : PDFs for the conformance probability and an acceptance region with PDFs for the global risks.

#### 4.4 Run calculations

Once all the input information are available, the button 'Run' enables to perform the required calculation. The result appears in the bottom right corner of the window and a graph summarizes the situation.

#### 4.5 Monte Carlo Markov Chain window

CAsoft holds a functionality for performing the simulation of Monte Carlo Markov Chains (MCMC). This is required only if the objective is to compute a conformance probability on the basis of a prior and a measurement distribution that do not constitute a couple of conjugate distributions.

#### 4.6 Export results

Once the calculations are performed, a button enables the user to export at the same time both the result and the input information that were used to get such result. This will generate an EXCEL file. The first sheet is dedicated to the input information while the second one will contain the results.

Such EXCEL file can also be used as an import file, by clicking the button 'Import file' at the top left of the main window. This enables the user





Table 1: Summary of available conjugate PDFs considered in CAsoft

Prior distribution	Measurement distribution	Posterior distribution
Gaussian $N(y_0, u_0)$	Gaussian $N(y_m, u_m)$	Gaussian $N(y_{Post}, u_{Post})$ $u_{Post} = \sqrt{\left(\frac{1}{u_0^2} + \frac{1}{u_m^2}\right)}$ $y_{Post} = u_{Post}^2 \left(\frac{y_0}{u_0^2} + \frac{y_m}{u_m^2}\right)$
Beta $B(\alpha, \beta)$	Binomial $Bin\left(N, \frac{X}{N}\right)$	Beta $B(\alpha + X, \beta + N - X)$
Gamma $G(\alpha, \beta)$	Poisson $Poiss\left(\lambda = \frac{Y}{N}\right)$	Gamma $G(\alpha + Y, \beta + N)$

to change some of the input information and to check the impact of the modifications on the results.

## 5 Methods

### 5.1 Objective of calculation

#### 5.1.1 Conformance probability

**Univariate case** In the case where no prior knowledge is available, the conformance probability is simply obtained using the Cumulative Distribution Function (CDF) of the measurement PDF. Let  $H(\eta_m)$  denote the CDF. Then, the conformance probability is equal to :

$$p_c = H(T_U) - H(T_L) \quad (1)$$

If prior knowledge is available, three cases of conjugate distributions are taken into account in CAsoft and are summarized in Table 1. In this case, equation 1 can also be used directly by considering with the corresponding posterior distribution.

In the case where the prior and likelihood distribution are not conjugate, then a Metropolis-Hastings algorithm (Figure 5) has to be used to generate Monte Carlo Markov Chain samples. However, the parametrization of such an algorithm is not straightforward. Some guidance is given in section 5.4 of this User's Manual.

**Bivariate case** In the bivariate case, the item is considered as conforming if both components are considered as conforming. Additionally, only rectan-

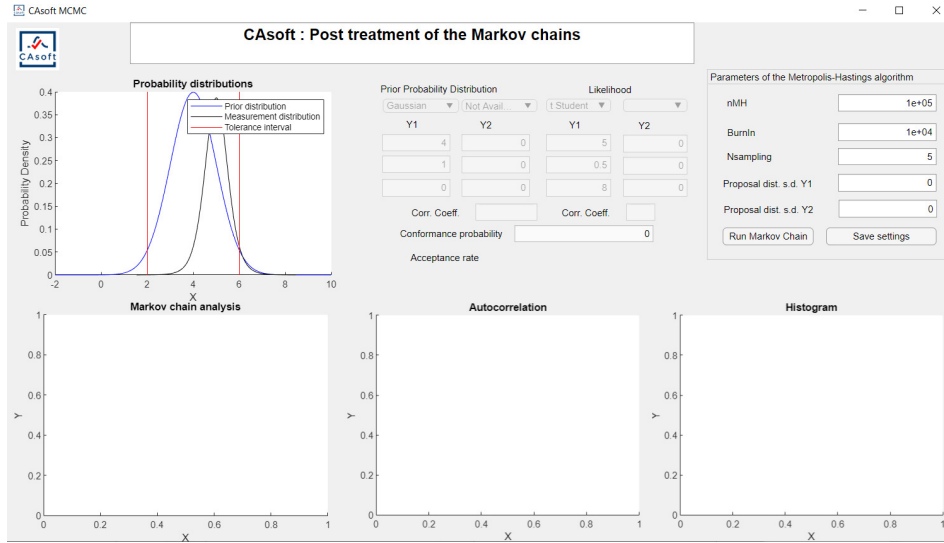


Figure 5: Screenshot of the parametrization of a Metropolis-Hastings algorithm in CAsoft

gular tolerance regions are considered in CAsoft. If the two components for the quantity are independent, then the "bivariate" conformance probability is equal to the product of the two marginal conformance probabilities.

**Case with multiple characteristics** This latter functionality is dedicated to users who need to evaluate quickly the conformity of multiple individual quantities, being considered separately (ie without the consideration of any correlation between the quantities). In this case, it is possible to consider as many quantities as required, through the software or a specific EXCEL spreadsheet. In order to make it simple, no prior knowledge can be specified and the only available PDFs for the measurement result are Gaussian or Student's t distributions. In this case, for each quantity  $Y_i$ , the conformance probability  $p_{c_i}$  is obtained as :

$$p_{c_i} = \int_{T_L}^{T_U} g_i(y_i) dy_i \quad (2)$$

where  $g_i(y_i)$  is the PDF for the quantity  $Y_i$ .



### 5.1.2 Acceptance interval

This functionality enables the user to calculate the optimal acceptance interval in order to satisfy a maximum value either for the consumer's risk or for the producer's risk. In the case of a two-sided tolerance interval, the limits of the acceptance interval are considered as symmetric from the tolerance limits, so that it is possible to calculate a guard band  $\omega$  which is the same for the lower values et the upper values.

### 5.1.3 Global risks

The global consumer's risk is defined as follows [3]:

$$R_c = \int_{-\infty}^{T_L} \left( \int_{A_L}^{A_U} g_0(\eta) h(\eta_m|\eta) \right) d\eta + \int_{T_U}^{\infty} \left( \int_{A_L}^{A_U} g_0(\eta) h(\eta_m|\eta) \right) d\eta \quad (3)$$

$$R_c = \int_{-\infty}^{T_L} g_0(\eta) \left( \int_{A_L}^{A_U} h(\eta_m|\eta) \right) d\eta + \int_{T_U}^{\infty} g_0(\eta) \left( \int_{A_L}^{A_U} h(\eta_m|\eta) \right) d\eta \quad (4)$$

Then, this function can be simplified a unique integral of a function of  $\eta$ , using the Cumulative Distribution Function (CDF)  $H_m(\eta_m|\eta)$  of the measurement distribution:

$$\int_{A_L}^{A_U} h(\eta_m|\eta) = H_m(A_u|\eta) - H_m(A_l|\eta) \quad (5)$$

$$R_c = \int_{-\infty}^{T_L} g_0(\eta) (H_m(A_u|\eta) - H_m(A_l|\eta)) d\eta + \int_{T_U}^{\infty} g_0(\eta) (H_m(A_u|\eta) - H_m(A_l|\eta)) d\eta \quad (6)$$

$$R_c = \int_{-\infty}^{T_L} GC_{aux}(\eta|A_l, A_u) d\eta + \int_{T_U}^{\infty} GC_{aux}(\eta|A_l, A_u) d\eta \quad (7)$$

The global producer's risk can also be evaluated thanks to a simple function of  $\eta$ . According to [3], the expression of the global producer's risk is :

$$R_p = \int_{-\infty}^{A_L} \left( \int_{T_L}^{T_U} g_0(\eta) h(\eta_m|\eta) \right) d\eta + \int_{A_U}^{\infty} \left( \int_{T_L}^{T_U} g_0(\eta) h(\eta_m|\eta) \right) d\eta \quad (8)$$



$$R_p = \int_{T_L}^{T_U} \left( \int_{-\infty}^{A_L} g_0(\eta) h(\eta_m|\eta) d\eta_m + \int_{A_U}^{\infty} g_0(\eta) h(\eta_m|\eta) d\eta_m \right) d\eta \quad (9)$$

$$R_p = \int_{T_L}^{T_U} g_0(\eta) \left( \int_{-\infty}^{A_L} h(\eta_m|\eta) d\eta_m + \int_{A_U}^{\infty} h(\eta_m|\eta) d\eta_m \right) d\eta \quad (10)$$

$$R_p = \int_{T_L}^{T_U} g_0(\eta) (H_m(A_L|\eta) - H_m(-\infty|\eta) + H_m(\infty|\eta) - H_m(A_U|\eta)) d\eta \quad (11)$$

Since  $H_m(-\infty|\eta) = 0$  and  $H_m(\infty|\eta) = 1$ :

$$R_p = \int_{T_L}^{T_U} g_0(\eta) (H_m(A_L|\eta) + 1 - H_m(A_U|\eta)) d\eta \quad (12)$$

## 5.2 Choice of probability distributions

The available PDFs in the software represent the situations that have been thought to be used in practice. As a result, not all combinations of PDFs are available. All the available choices for both distributions are summarized in table 2.

## 5.3 Visualization of results

CASoft provides also graphs to help in the interpretation of the results when it is possible. The sections below provide an example of how the results are represented for each kind of objective.

### 5.3.1 Conformance probability

Figure 6 and Figure 7 represent respectively the output from a calculation of a conformance probability leading to either a decision of acceptance (and a specific consumer's risk) or a decision of rejection (and a specific producer's risk).



Table 2: Summary of available PDFs in CAsoft

Prior distribution	Measurement distribution
Beta	Binomial
Gaussian	Gaussian
	t-Student
	Lognormal
Rectangular	Gaussian
	t-Student
	Lognormal
Gamma	Gaussian
	t-Student
	Lognormal
	Poisson
Lognormal	Gaussian
	t-Student
	Lognormal

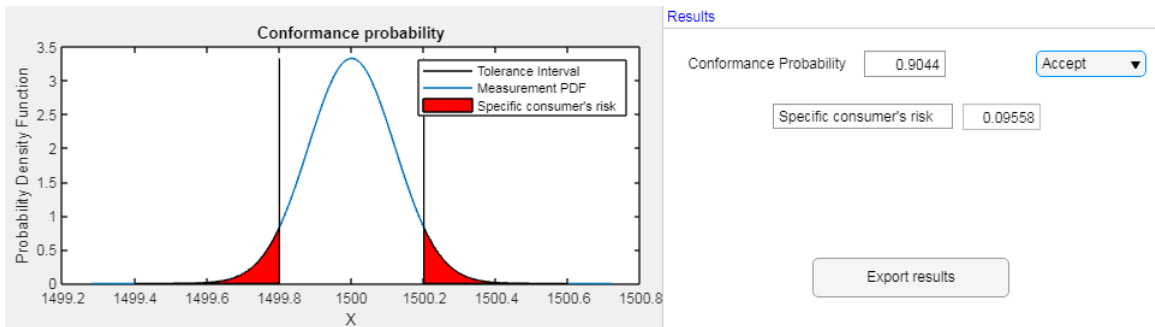


Figure 6: Screenshot of visualization of the results for the calculation of a specific consumer's risk

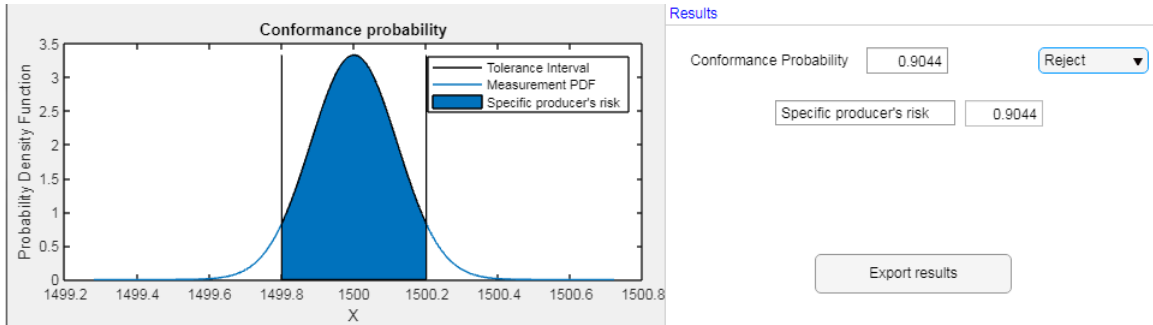


Figure 7: Screenshot of visualization of the results for the calculation of a specific producer's risk

### 5.3.2 Global risks

When global risks are computed, the graph provides a visualization of the tolerance interval on the X-axis and of the acceptance interval on the Y-axis. True values are simulated from the prior distribution and corresponding measured values are simulated from the likelihood distribution. These simulated values are represented on the graph with three colors:

- blue : The decision is correct (conforming items accepted or non-conforming items rejected)
- green : Consumer's risk (non-conforming items accepted)
- red : Producer's risk (conforming items rejected)

An example of such representation is given in Figure 8.

### 5.3.3 Acceptance interval

The representation for the calculation of an acceptance interval is the same as the one for the global risks. However, on the right, the proposed bounds of the acceptance interval are given, with information about the corresponding band guard.

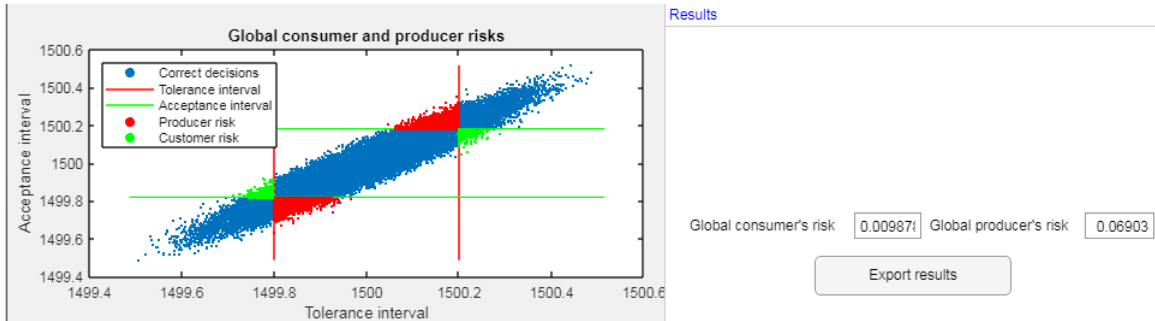


Figure 8: Screenshot of visualization of the results for the calculation of global risks

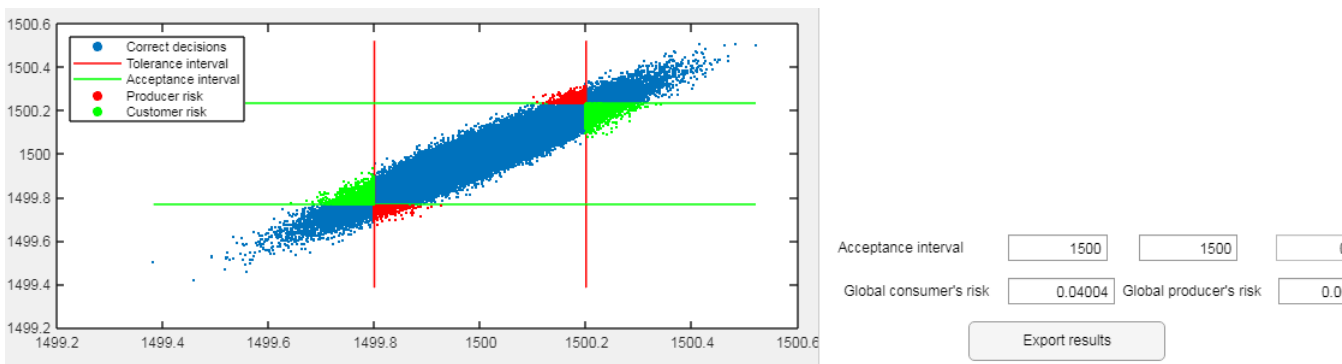


Figure 9: Screenshot of visualization of the results for the calculation of an acceptance interval



## 5.4 Guidance on the parametrization of a Metropolis-Hastings algorithm

### 5.4.1 Principle

A Metropolis-Hastings algorithm enables to sample from a posterior distribution when the prior and likelihood distribution are not conjugate. In other words, it has to be used when this posterior distribution cannot be determined analytically. It is an Accept/reject method, which consists in the simulation of a plausible value for the parameter, which is sampled from the prior distribution for the first iteration, and from a so-called proposal distribution centred on the previous value of the parameter in the Markov Chain. In CASoft, the proposal distribution is chosen to be a Gaussian distribution and the user is allowed to personalize the standard deviation of this distribution, expressed as a fraction (between 0 and 1) of the previous value in the chain. The higher the standard deviation, the further the new proposed value is likely to be from the previous value in the chain.

### 5.4.2 Parameters

The MH algorithm proposed in CASoft allows the user to control 4 parameters :

- The total sample size (nMH)
- The number of first discarded values in the chain, often called "Burn-In"
- The sampling rate (Nsampling), so that the algorithm only considers in the final chain only 1 in Nsampling values from the total chain.
- The relative standard deviation of the proposal distribution

### 5.4.3 Optimization of the algorithm

A full characterization of the convergence of the Markov Chain could not be implemented due to a limited development time in the framework of the EMPIR 17SIP05 CASoft project. However, three information can be very useful to determine the optimal set of parameters for a given case :





- The acceptance rate is the rate of accepted proposed values among the nMH iterations. A low acceptance rate (close to zero) means that the proposed values are likely to land in regions of low probability, resulting in a slow convergence for the chain. A high acceptance rate (close to 1) means that the exploration of the space is very slow, resulting also in a slow convergence of the chain. As a result, a convergence rate 0.30 is often considered to be a good compromise.
- The graphs in the bottom left represents the evolution of the Markov chain. Such a graph can be used to evaluate whether the Markov Chain can be considered to have converged or not. Please notice that it is only provided as an indication. For a more rigorous evaluation of the convergence, the user should consider to apply a dedicated methodology such as the procedures provided by Gelman & Rubin [2] or Brooks & Gelman [1] that are not available within CAsoft.
- The graph in the bottom center provides a representation of the auto-correlation graph of the chain. This should be used to determine the optimal value of  $N_{sampling}$ . Common practice is to determine the order as the one at which the slope is significantly different.

## 6 Examples

### 6.1 Conformance probability : Zener diode (JCGM106:2012, 7.3, Example 1)

The breakdown voltage  $Vb$  of a Zener diode is measured, yielding a best estimate  $vb = 5.47\text{ V}$  with associated standard uncertainty  $u = 0.05\text{ V}$ . Specification of the diode requires  $Vb \leq 5.0\text{ V}$ , which is an upper limit on the breakdown voltage. The corresponding result is represented in Figure 10.

### 6.2 Global risk : Precision resistors (JCGM106:2012, 9.5.3.2)

A supplier of electrical components produces wire-wound precision resistors of nominal resistance  $1500\ \Omega$ . For each resistor (the item), the resistance  $Y$  (the property of interest) is specified to lie in a tolerance interval defined by the limits  $T_L = 1499.8\ \Omega$  and  $T_U = 1500.2\ \Omega$ .

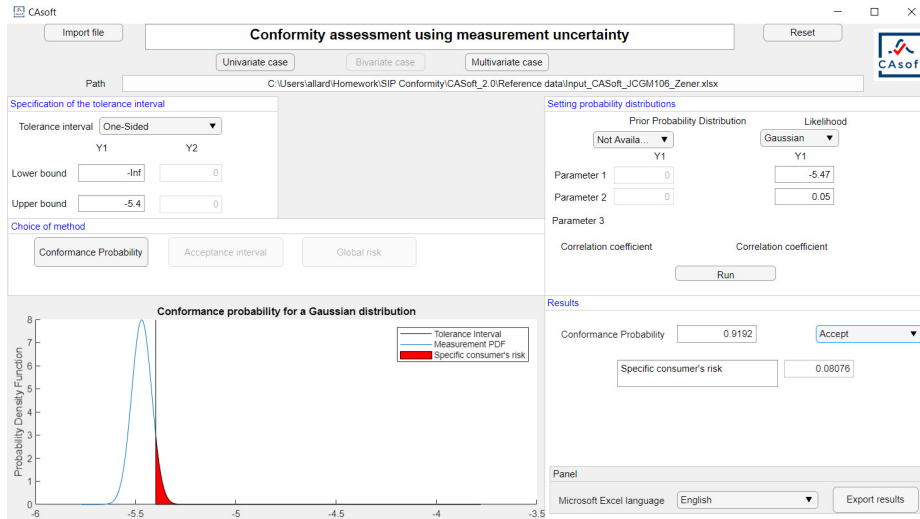


Figure 10: Zener diode example with CASoft

A machine for producing such resistors is evaluated by measuring a sample of its output, using a high-accuracy ohmmeter with negligible measurement uncertainty. A histogram of the measured values appears normal in shape, centred on the nominal value with a standard deviation  $\sigma = 0.12 \Omega$ . Based on this information, a normal PDF  $g_0(\eta)$  with mean value  $y_o = 1500 \Omega$  and standard deviation  $u_0 = \sigma = 0.12 \Omega$  is assigned to model the production process. In production, the resistors are inspected for conformity with specification using a calibrated high-speed ohmmeter. A normal PDF with  $u_m = 0.04 \Omega$  is assigned.

In order to reduce the probability of shipping resistors that do not meet specification (the consumer's risk), acceptance limits  $A_L = 1499.82$ ,  $A_U = 1500.18$  are chosen inside the tolerance interval, creating a guarded acceptance interval with symmetric guard bands. Figure 11 represents the result for this problem. As a consequence, the global consumer's risk is 1% and the global producer's risk is 6.9%.

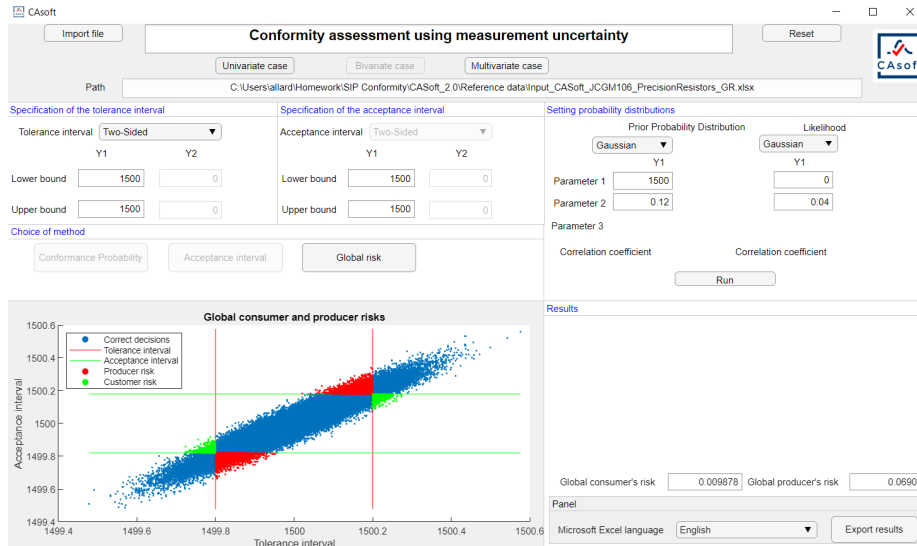


Figure 11: Precision resistors example with CASoft

### 6.3 Acceptance interval : Adaptation from Precision resistors

Using data from Example 2, it is expected that the global producer's risk does not exceed 5%. CASoft enables to specify such targeted value for the producer's risk as in Figure 12.

CAsoft computes the optimal interval as [1499.809; 1500.191], which is the interval that minimizes the global consumer's risk and satisfies to the maximum admissible value for the producer's risk.

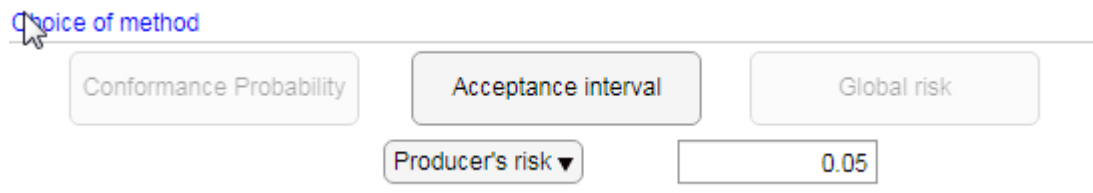


Figure 12: Specification of a targeted risk value for the calculation of the associated acceptance interval



## 7 Acknowledgement

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